Problem 8

To disprove that SAW is context-free, we can first assume that it is a context-free language.

And the string w be a string with N^p E^p S^2p+1 W^2p+1 N^2p+1 E^p S^p+1 E^p where p= 2k

A graph paper with a drawing of a square and a square with a red arrow

Description automatically generated

Also, we need to assume that the string w is satisfied with these conditions:

1. where for i >= 0, u(v^i)x(y^i)z
2. |vwx|<=2^k
3. And vy are not empty string.

Case 1 Consider either v or y in the portion of each section.

1. For string with v or y in range N. For example, when a string of N^p E^p S^2p+1 W^2p+1 N^2p+1 E^p S^p+1 E^p

When we pump N with i=2 where N^p+1 E^p S^2p+1 W^2p+1 N^2p+1 E^p S^p+1 E^p it will not be a self-avoiding walk which contradicts the context-free language of SAW as the N string will have the same length as the S string portion, then E string portion will collide with N string portion.

1. For string with v or y as E. For example, when a string of N^p E^p S^2p+1 W^2p+1 N^2p+1 E^p S^p+1 E^p

When we pump the E portion down to I =0, we will get the last E portion to collide with the S portion. This contradicts the CFL of SAW since it is not a self-avoiding walk.

1. For string with v or y as S. For example, when a string of N^p E^p S^2p+1 W^2p+1 N^2p+1 E^p S^p+1 E^p

When we pump the S portion down to i=0, we will get the last E portion to collide with the N portion. This contradicts the CFL of SAW since it is not a self-avoiding walk.

1. For string with v or y as W. For example, when a string of N^p E^p S^2p+1 W^2p+1 N^2p+1 E^p S^p+1 E^p

When we pump the W portion down to I =0, we will get the last E portion to collide with the S portion. This contradicts the CFL of SAW since it is not a self-avoiding walk.

1. For string with v or y as N. For example, when a string of N^p E^p S^2p+1 W^2p+1 N^2p+1 E^p S^p+1 E^p

When we pump N with i=2 where N^p E^p S^2p+1 W^2p+1 N^2p+2 E^p S^p+1 E^p it will not be a self-avoiding walk which contradicts the context-free language of SAW. After pumping the N the E portion will collide with the first N portion.

1. For string with v or y as E. For example, when a string of N^p E^p S^2p+1 W^2p+1 N^2p+2 E^p S^p+1 E^p

When we pump E with i=2 where N^p E^p S^2p+1 W^2p+1 N^2p+2 E^p S^p+1 E^p it will not be a self-avoiding walk which contradicts the context-free language of SAW. After pumping the N the S portion will collide with the first N portion.

1. For string with v or y as S. For example, when a string of N^p E^p S^2p+1 W^2p+1 N^2p+1 E^p S^p+1 E^p

When we pump the S portion down to i=0, we will get the last E portion to collide with the N portion. This contradicts the CFL of SAW since it is not a self-avoiding walk.

1. For string with v or y as E. For example, when a string of N^p E^p S^2p+1 W^2p+1 N^2p+1 E^p S^p+1 E^p

When we pump the E portion to i=2, the E portion will collide with the S portion and it will not be a self-avoiding walk which contradicts the SAW being a CFL.

Case 2 either v or y contains two consequences of direction.

1. For string with v or y contains NE, ES,   
   a. N^pE^p S^2p+1 W^2p+1 N^2p+1 E^p S^p+1 E^p,  
   b. N^p E^pS^2p+1 W^2p+1 N^2p+1 E^p S^p+1 E^p  
   c. N^p E^p S^2p+1 W^2p+1 N^2p+1E^p S^p+1 E^p   
   d. N^p E^p S^2p+1 W^2p+1 N^2p+1 E^pS^p+1 E^p.

While pumping it i=2 we will get   
N^p+1E^p+1 S^2p+1 W^2p+1 N^2p+1 E^p S^p+1 E^p   
N^p E^p+1S^2p+2 W^2p+1 N^2p+1 E^p S^p+1 E^p   
N^p E^p S^2p+1 W^2p+1 N^2p+2E^p+1 S^p+1 E^p  
N^p E^p S^2p+1 W^2p+1 N^2p+1 E^p+1S^p+2 E^p  
which will not be a self-avoiding walk since the S portion will collide with the N portion.

1. For string with v or y contains SW and WN,   
   N^p E^p S^2p+1W^2p+1 N^2p+1 E^p S^p+1 E^p   
   N^p E^p S^2p+1 W^2p+1N^2p+1 E^p S^p+1 E^p.

While pumping it down to i=0, in these two situations the collision will occur on the S portion it will collide with the N portion.

1. For string with v or y contains SE,  
   N^p E^p S^2p+1 W^2p+1 N^2p+1 E^p S^p+1E^p

While pumping it down to i=0, the E portion will collide with the N portion which will not be a self-avoiding walk.

For all the cases, we can conclude that it contradicted the assumption that we made earlier where SAW is a context-free language but by proving by using pumping lemma.

In conclusion, SAW is not a context free language.